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Tsallis $p_\perp$ distribution from statistical clusters

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A R T I C L E I N F O

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A B S T R A C T

It is shown that the transverse momentum distributions of particles emerging from the decay of statistical clusters, distributed according to a power law in their transverse energy, closely resemble those following from the Tsallis non-extensive statistical model. The experimental data are well reproduced with the cluster temperature $T \approx 160$ MeV.

1. It is now well-documented that the transverse momentum distribution of various particles at high energy and in a very broad range of the transverse momentum is correctly described by the Tsallis distribution. This was observed in all high-energy experiments [1–4], as well as in the recent phenomenological analyses [5–10]. This observation is usually interpreted in terms of the statistical model of particle production, employing the Tsallis non-extensive statistics [11,12]. Such interpretation,[1] although very attractive, meets a serious difficulty, however: it is indeed not easy to explain why any statistical model can apply at very large transverse momenta, where the perturbative QCD phenomena are known to dominate.

This problem was recently addressed in a series of papers [14, 15] where the ideas derived from perturbative QCD (as applied to hard interactions), accompanied by the parton cascade (responsible for the jet fragmentation) were used to explain this puzzling result.

In this note, following the general idea suggested in [14,15] (cf. also [16]), we apply it to the statistical model of particle production which is rather successful in describing data on particle multiplicities (for a review, see, e.g. [17,18]). We show that the Tsallis formula can be recovered, to a good accuracy, in the model where the observed particles are decay products of clusters [19–21] which (i) decay according to the standard Boltzmann statistics, and (ii) the distribution of their Lorentz factors follows a power law, as suggested by (perturbative) QCD. This observation indicates that the experimental validity of the Tsallis formula may be interpreted as another confirmation of the standard statistical model rather than that of its non-extensive Tsallis version.[2]

Indeed, the intriguing "unreasonable" success of the statistical model in description of multi-particle production in various processes and at various energies suggests that the final stage of the process of hadronization is dominated by the hadrons in the state of statistical equilibrium. It is also clear that the equilibrium cannot be global, as the observed spectra are far from isotropic. These observations lead naturally to the idea [19] that the transition from the early state of the process, dominated by interactions between the hadronic constituents, most likely proceeds through an intermediate stage of clusters emitting the final hadrons according to the rules of statistical physics.

If one admits that this process of cluster formation and thermal decay is a universal feature of hadronization, one is led to the conclusion that also the high transverse momentum jets hadronize in the same way (cf. [20]). It follows that the characteristic features of clustering should leave their imprints even in the region of hard physics. In the present paper we show that this picture, when combined with the power law distribution of the (transverse) Lorentz factor of the cluster, leads to transverse momentum distribution of the decay products which is very close to that of Tsallis (and thus also close to experiment).

In the next section the idea of the statistical cluster is formulated and the transverse momentum distribution of its decay products is derived. The relation to the Tsallis distribution is dis-

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[1] The fundamental relation between the Boltzmann and Tsallis statistical models is clearly explained in [13].

[2] Another approach aiming at the explanation of the power law tails within statistical model is discussed in [22].

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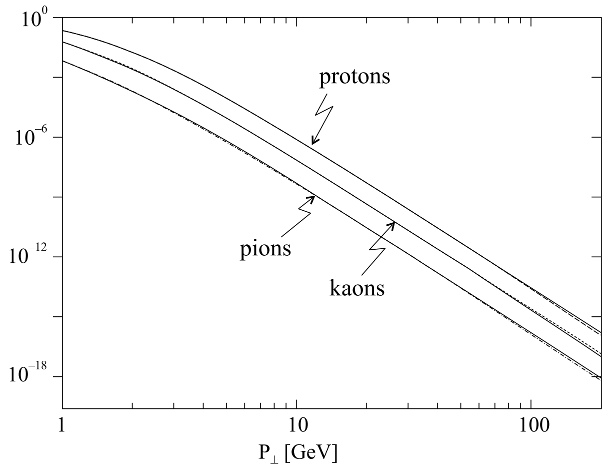

Fig. 1. Transverse momentum distribution of pions, kaons and protons from the statistical cluster decay (dashed lines), normalized to 1 at $p_\perp = 0$, compared to two Tsallis distributions (Eq. (7)) (full lines). 1 GeV $\le p_\perp \le$ 200 GeV. $T = 155$ MeV, $\kappa = 6.5$. Best fit from $p_\perp = 0$ to $p_\perp = 50$ GeV.

cussed in Section 3. Summary and comments are given in the last section.

2. Following the ideas explained above, the decay distribution of the statistical cluster at rest is taken in the form of the Boltzmann distribution which, for a cluster moving with the four-velocity u^μ becomes

$$\rho(p;u)d^2p_\perp dy = e^{-\beta p_\mu u^\mu} d^2p_\perp dy \tag{1}$$

where $\beta = 1/T$.

Consider a cluster at rapidity Y moving in the transverse direction with the velocity $v_\perp$. We have

$$u_0 = \sqrt{1+u_\perp^2}\cosh Y; \quad u_z = \sqrt{1+u_\perp^2}\sinh Y; \quad v_z = \tanh Y;$$
$$u_\perp = \gamma v_\perp; \quad \gamma = (1-v^2)^{-1/2} \to \sqrt{1+\gamma^2 v^2} = \gamma. \tag{2}$$

The distribution of particle momentum is then

$$\rho(p,y)dy = dyd^2p \int d^2v_\perp dY G(v_\perp, Y) e^{-\beta\gamma_\perp m_\perp \cosh(y-Y) - \beta p_\perp u_\perp \cos\phi} \tag{3}$$

where ϕ is the angle between v and $p_\perp$ and where we have denoted

$$\gamma_\perp \equiv \sqrt{1+u_\perp^2} = \gamma\sqrt{1-v_z^2} \to u_\perp = \sqrt{\gamma_\perp^2 - 1} \tag{4}$$

Integration over ϕ and y gives the distribution of the transverse momentum:

$$\rho(p)d^2p = d^2p \int d^2v_\perp dY G(v_\perp, Y) K_0[\beta m_\perp \gamma_\perp] I_0[\beta p_\perp u_\perp] \tag{5}$$

3. To evaluate the distribution of transverse momenta of the cluster decay products, one needs the distribution of the cluster transverse velocity $v_\perp$. In this paper we study a power law in the transverse Lorentz factor $\gamma_\perp$ (for a fixed cluster mass, this would correspond to a power law in its transverse energy). Thus we take

$$d^2v_\perp dY G(v_\perp, Y) \sim G(Y) dY \gamma_\perp^{-\kappa} d\gamma_\perp \tag{6}$$

Given simplicity of this assumption, it was rather surprising to find that it leads to the distribution which closely resembles that of

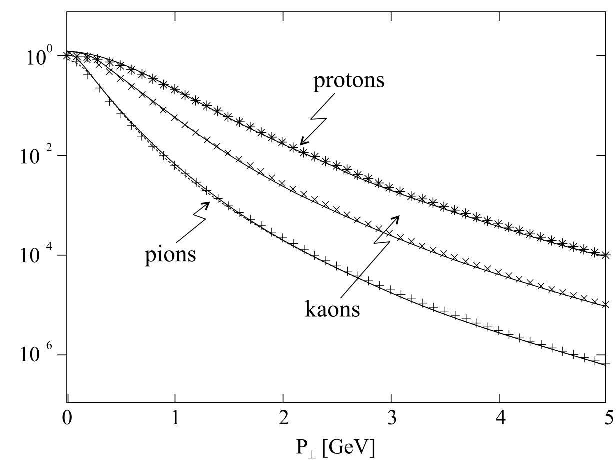

Fig. 2. Same as Fig. 1 but for $0 \le p_\perp \le 5$ GeV. Lines: the Tsallis distribution. Crosses and stars: statistical clusters.

Tsallis,[3] from $p_\perp \approx 100$ MeV up to $p_\perp = 200$ GeV. This was verified numerically for the cluster temperature in the region from 100 till 180 MeV and the power κ from 4 till 7, i.e. in the range covering the physical conditions one may expect in high-energy collisions.

An example of such calculation is shown in Figs. 1 and 2 where the distributions of pions, kaons and protons evaluated using (5) and (6) with $\kappa = 6.5$ and $T = 155$ MeV, are compared with the two versions of the Tsallis distribution [6,8,13,23]:

$$D_1 = cm_\perp[1+(q-1)m_\perp/T_{ts}]^{q/(1-q)};$$
$$D_2 = c[1+(q-1)m_\perp/T_{ts}]^{1/(1-q)}, \tag{7}$$

where c is the normalization constant, $q-1$ measures the deviation from the standard statistical model and T_{ts} is the Tsallis temperature.[4]

One sees that, except at very small $p_\perp$, below ~100 MeV, there is an excellent agreement between the two formulations and for all kinds of particles. One also sees that for $p_\perp \ge 1$ GeV it is difficult to distinguish between the two versions of the Tsallis distributions. For the distribution D_1 the Tsallis parameter T_{ts} can be approximated by the simple relation $T_{ts} \approx (q-1)T$. This is not true, however, for D_2. In this case the relation between T_{ts} and T is more complicated and, moreover, it depends substantially on the particle mass.

Recently, a new analysis of transverse momentum distribution of charged particles in terms of the Tsallis distribution has been published [6]. To compare these results with our approach, we have evaluated the distribution following from the decay of a cluster for pions, kaons and protons and constructed the distribution of charged particles, using the weights (1:1:2), as proposed in [6]. In Fig. 3 the results in the region from $p_\perp = 0$ till $p_\perp = 5$ GeV are compared with the Tsallis distribution from [6]. One sees that the agreement is very good, except at $p_\perp < 100$ MeV. The parameters of the Tsallis distribution in this case are $q-1 = 0.150$ and $T_{ts} = 76$ MeV, in good agreement with [6]. The region $p_\perp \ge 5$ GeV is not shown because in this region one simply cannot distinguish between the two curves.

[3] Qualitatively, the result of this kind may be actually expected, as it is well known [13,23,26] that the Tsallis formula is naturally obtained by adequate fluctuations of the parameters of the Boltzmann spectrum.

[4] The form D_1 is obtained by demanding maximum of the Tsallis entropy, i.e. thermodynamic equilibrium [24,25]. The second form is the standard Tsallis distribution.

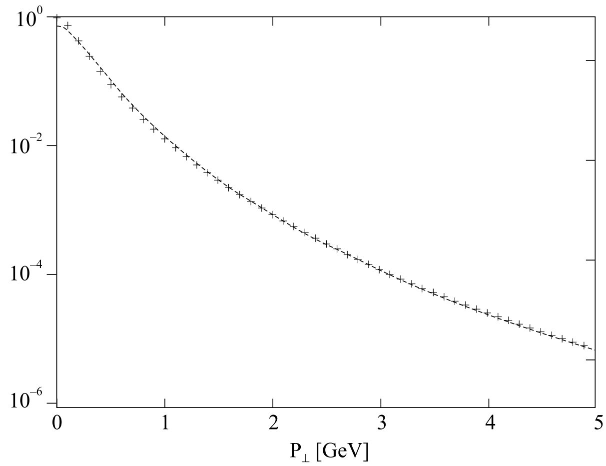

Fig. 3. Transverse momentum distribution of charged particles from the statistical cluster decay (crosses), compared to the Tsallis distribution (dashed line) used in [6] (the first formula in (7)). $0 \le p_\perp \le 5$ GeV. $T = 155$ MeV, κ=6.5. Best fit from $p_\perp = 0$ to $p_\perp = 50$ GeV.

4. In summary, we have discussed the transverse momentum distributions of particles emitted in the decay of a statistical cluster. It was shown that if the (transverse) Lorentz factor of the cluster follows a power law, the resulting distribution is very close to that derived from the Tsallis non-extensive statistics.

This result may be considered as a possible explanation of the surprising observation that the Tsallis formula works not only at small transverse momenta (where the ideas of statistical equilibrium may be applicable) but even at transverse momenta as large as ∼200 GeV.

Some comments are in order.

(i) It should be emphasized that the observed similarity between the Tsallis formula and that following from the statistical cluster decay, is only an approximation. Our results indicate, however, that it may be rather difficult to distinguish experimentally between these two approaches. Perhaps the measurements at larger transverse momenta may be helpful, as the two distributions start to deviate from each other at energies above 200 GeV.

(ii) We have been discussing emission of a single statistical cluster. As it is rather unlikely that a high-energy jet may fragment into a single cluster, production of many clusters must also be considered. Since our discussion concerns only the single-particle distribution, however, the results are insensitive to the number of clusters produced in a given event, provided they are emitted independently.

(iii) Clearly, the power law assumed in (6) is only a phenomenological guess and should be treated as such. Its main advantage is the extreme simplicity (for more elaborate calculations see, e.g., [14–16]). Needless to say, the parameter κ remains free at the present stage, and cannot be reliably evaluated from theory.

(iv) It has been shown recently [27] that the distributions of transverse momenta at various energies follow a scaling law, suggested by the saturation property of the parton distributions. An interpretation of this observation in terms of the Tsallis approach was proposed in [16,28]. It would be thus interesting to investigate how this scaling property of the spectra translates into the results shown in the present paper.

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